



Searching the musical rehearsal room

Jens Holger Rindel

Multiconsult, Oslo, Norway, jehr@multiconsult.no

The room dimensions are important for the frequency distribution of the normal modes of the room. The influence of the dimension ratio is analysed in box-shaped rooms with volume between 25 m³ and 300 m³. Three different criteria have been applied to evaluate whether the frequency distribution is favourable; a smooth frequency response, the variance of the interval between modal frequencies, or the number of tones in the musical scale, supported by at least one of the room modes. Clearly, a square room or a cubic room are unfavourable and should be avoided as a music room. The results of the applied methods agree that there are three usable optimum dimension ratios, which have also been reported previously in the literature. The optimum ratios are (1:1.2:1.45), (1:1.4:1.89), and (1:1.48:2.12). However, it is also clear that nearly optimum dimension ratios are found within a certain range around each optimum. The analysis leads to a practical method for choosing favourable room dimensions in a music room.

1 Introduction

Singing or playing a musical instrument in a room is greatly affected by the acoustics of the room. We can say that the room gives support to the sound. In fact, the room behaves acoustically as an extension of the musical instrument. This is not only a matter of reverberation time and volume, but also concerns the frequency distribution of the room modes. The latter is particularly important in small rooms like music practice rooms and rehearsal rooms for small ensembles. In a small room, there are not many room modes in the low frequency range and the room support can be very unequal for the different musical tones. Thus, this study is restricted to box-shaped rooms with volumes between 25 m³ and 300 m³.

Musical instruments have been developed and improved over centuries and crafted to produce sound of high quality over the entire tonal range of the instrument. One of the challenges of the instrument makers has been to obtain equal tone quality for every semitone. Therefore, the extension of the instrument (the room) should also be designed with the best possible acoustical quality.

As early as 1900, Sabine [1] commented on the question of room dimension ratios: “Thus the most definite and often repeated statements are such as the following, that the dimensions of a room should be in the ratio 2:3:5, or according to some writers, 1:1:2, and others, 2:3:4; it is probable that the basis of these suggestions is the ratio of harmonic intervals in music, but the connection is untraced and remote.” Sabine was very sceptical to such suggestions.

In 1942, Volkman [2] suggested different ratios based on $2^{1/3}$ and presented a diagram with recommended ratios for different room sizes, e.g. 1:1.25:1.6 for small rooms and 1:1.6:2.5 for average sized rooms. An early scientifically based study of room dimension ratios was published in 1946 by Bolt [3]. He used the frequency spacing between the modes in the low-frequency range to calculate a frequency spacing index and presented a graph with a region for ‘good’ dimension ratios. Since then, there have been a vast number of publications on the topic, all of which shall not be referred here. Walker [*] applied the mean-square spacing of the mode frequencies as a room quality index. He looked at frequencies up to 120 Hz and typical room volumes of 100 and 200 m³. He suggested upper and lower limits as $l/h \geq 1.1$ w/h , and $l/h \leq 4.5$ $w/h - 4$. Cox and D’Antonio [4] applied an image source model with source in one corner and receiver in the opposite corner to calculate the frequency response. By numerical optimisation the room dimensions were changed to achieve the flattest possible frequency response in the frequency range 20 Hz - 200 Hz. They found the worst-case ratio to be (1:1.075:1.868), but they did not report the optimized dimension ratios. The same idea was applied recently by Meissner [5], who reported very detailed results.

The problem studied in this paper is, how to choose the room dimensions to obtain a room with the best possible acoustic support for playing musical instruments. Three different criteria for this are applied in the following.

2 Normal modes in a rectangular room

A rectangular room with room dimensions L , W , and H as shown in Figure 1, have normal modes with frequencies f_n calculated with the formula:

$$f_n = \frac{c}{2} \sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_y}{W}\right)^2 + \left(\frac{n_z}{H}\right)^2} \quad (1)$$

Here, c is the speed of sound in air ($c = 343.3$ m/s at 20°C) and the modal numbers are (n_x, n_y, n_z) . It is assumed that $L \geq W \geq H$.

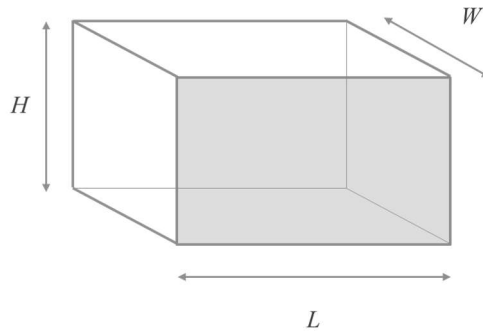


Figure 1: Rectangular room with main dimensions.

3 Smoothness of the frequency response

The first criterion for evaluation of the distribution of the room modes is to look at the frequency response between 20 Hz and 200 Hz. Since the frequency response (or transfer function) depends strongly on the position of source and receiver in the room, the global frequency response is considered. This is obtained with the source and receiver in opposite corners, and this ensures all room modes to be included in the frequency response. Figure 2 shows an example of a calculated global frequency response using the modal energy analysis model by Rindel [6, 7].

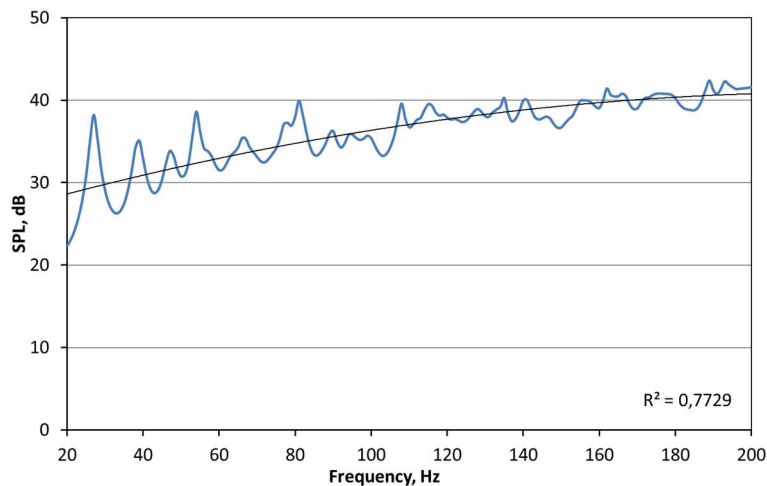


Figure 2: Calculated global frequency response from 20 Hz to 200 Hz of an 85 m^3 room with dimensions $6.36 \text{ m} \times 4.44 \text{ m} \times 3.00 \text{ m}$. All surfaces have the absorption coefficient $\alpha = 0.20$. The black line is the best-fit regression line for a 2nd order polynomial.

The smoothness of the frequency response was used as a criterion by Meissner [5], who compared the frequency response with a 2nd order polynomial and used the normalised correlation coefficient as criterion for the smoothness. With this method he could produce graphs like those shown in Figure 3. A drawback that makes this method a little complicated is, that the results depend not only on the room dimension ratio, but also on the absolute volume of the room and on the absorption coefficients of surfaces. Higher absorption means more shallow room modes with increased bandwidth and thus increases smoothness. A small volume means more focus on the lowest room modes, which naturally are more separated than the higher room modes. A large room volume means that the lowest room modes are well below 20 Hz and thus not within the frequency range being analysed.

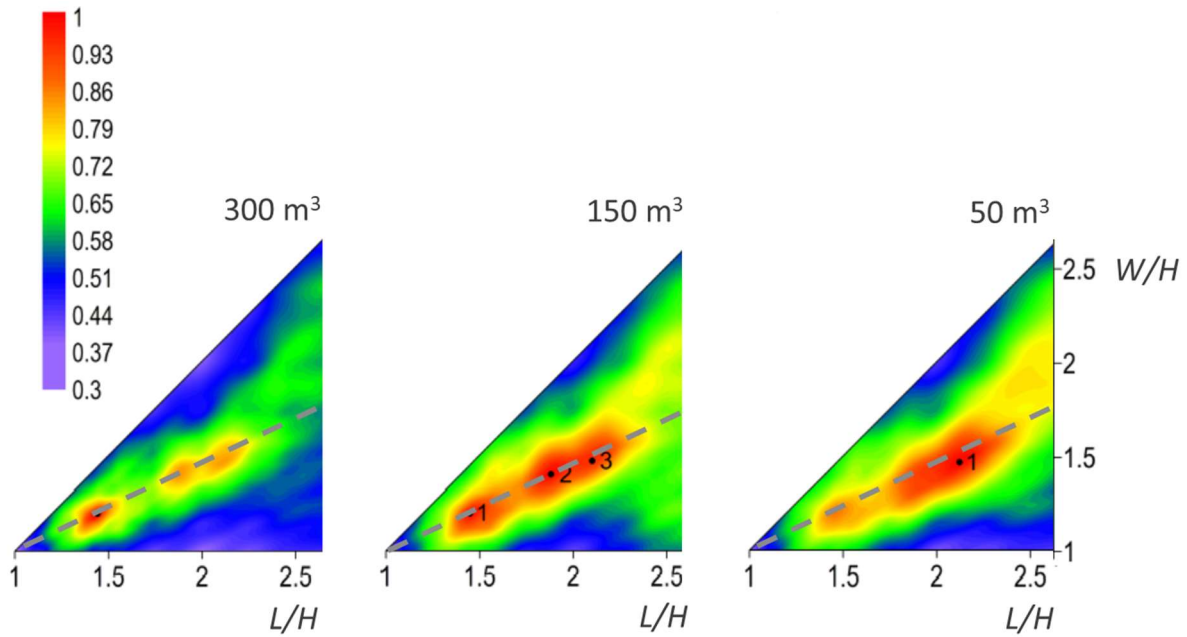


Figure 3: Degree of smoothness of frequency response as function of dimension ratio in three different room volumes. Source and receiver are in opposite room corners and all surfaces have the absorption coefficient $\alpha = 0.20$. The colour scale indicated the degree of smoothness with 1 as maximum. The dashed line is the linear regression line given in Equation (2). Based on figures adapted from Meissner [5].

The results shown in Figure 3 are for three different room volumes, 50 m³, 150 m³ and 300 m³. The following dimension ratios are found to produce very smooth frequency responses: A (1:1.20:1.45), B (1:1.40:1.89), and C (1:1.48:2.12). The three rooms are shown in isometric view in Figure 4.

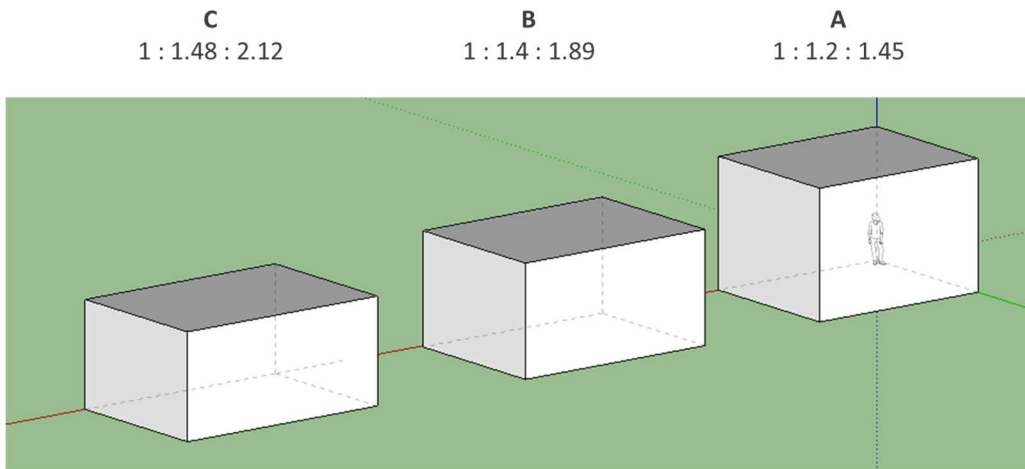


Figure 4: Isometric view of three rooms with dimension ratios A, B and C, respectively.

The optima marked with numbers in Figure 3 are A in the 300 m³ room, C in the 50 m³ room, and all three optima in the 150 m³ room. In the original paper by Meissner [5], a second optimum was found in the 50 m³ room, (1:2.55:3.44). However, this is not usable in practice because the room height would be only $H = 1.8$ m, and thus this result is not included here. The optimum C is realistic in a 50 m³ room, as it leads to the room height $H = 2.6$ m.

From the graphs in Figure 3 it is interesting to observe that the “nearly-optimal” areas (red and orange) form a ridge along a line through the three points A, B, and C. The three points are approximately on a straight line, and the equation for the regression line is:

$$L = 2.3558 \cdot W - 1.3838 \cdot H \quad (R^2 = 0.996) \quad (2)$$

4 Number of musical tones supported by room modes

The second criterium for evaluation of the distribution of the room modes is to look at the tones produced by musical instruments. Figure 5 shows the keys of a common piano being a standard instrument in many music rehearsal rooms.

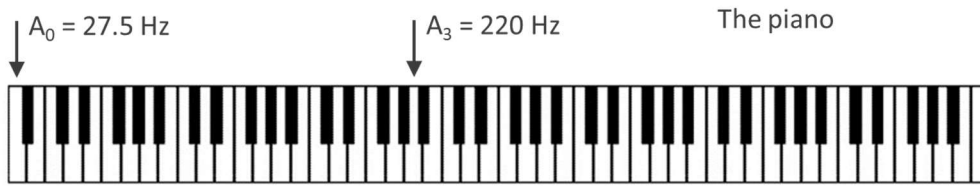


Figure 5: The keys of a common piano. The lowest three octaves from A_0 to A_3 are considered for the present analysis.

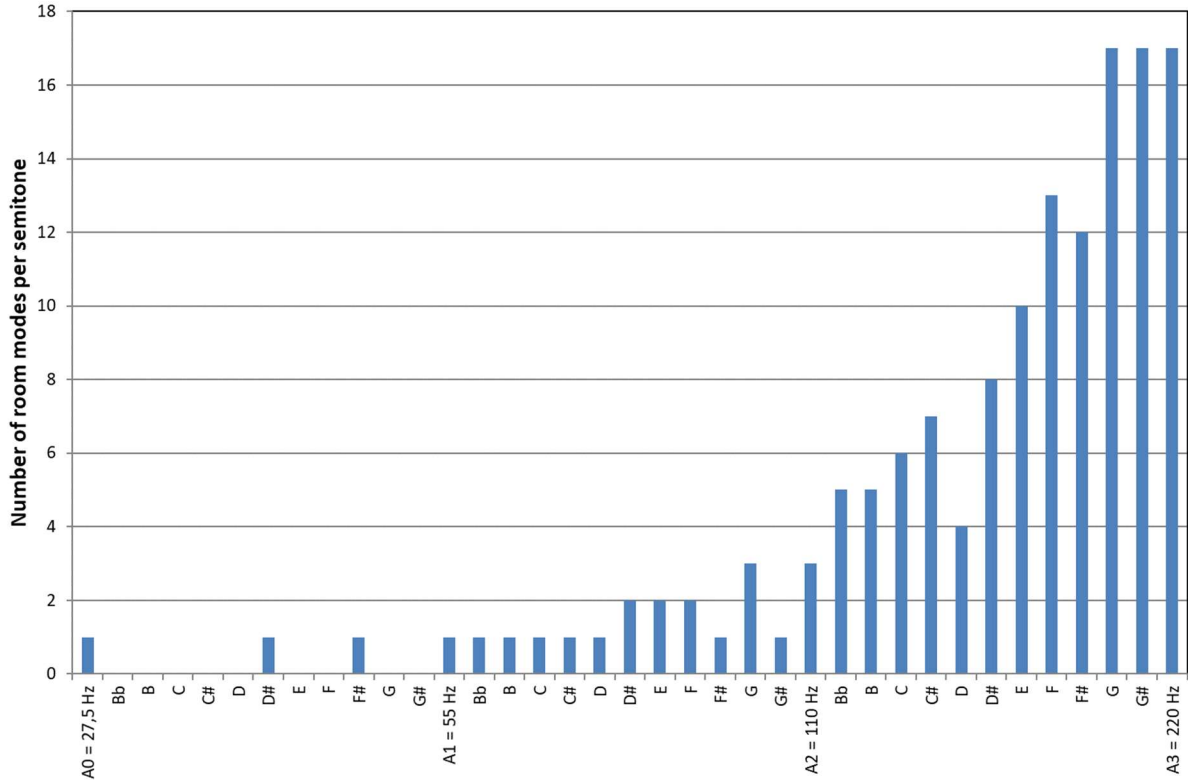


Figure 6: Number of modes per semitone from A_0 to A_3 in the example room. Nine of the tones are not supported by a room mode, 28 out of 37 tones are supported.

Since the density of the room modes increases strongly with frequency, it is only the low frequency range that need to be analysed. This is chosen to be the three lowest octaves on the piano covering 37 semitones from $A_0 = 27.5$ Hz to $A_3 = 220$ Hz. The method is to count the number of room modes for each musical tone (plus/minus a quarter tone). The result

is the number of tones covered by one or more room modes, and this number should be as large as possible. The maximum number is 37, and this can only be reached in rooms with a volume of 1000 m³ or more. This method was introduced by Rindel [6].

The example room of 85 m³ used previously in Figure 2 is used again for showing the number of semitones supported by room modes, see Figure 6. This room has dimensions as room C in Figure 4, and for a room of this rather small size the coverage of musical tones is good (28 of 37).

Figure 7 shows the results when the room dimension ratios are varied, in this case for a 150 m³ room. The best results are found in roughly three zones that are centered around the previously found optima A, B, and C. The worst results are for the cubic room and the (1,1,2) and (1,2,2) cases.

L/H	1	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9	1.95	2	2.05	2.1	2.15	2.2	W/H	
	22	25	26	27	28	27	27	25	25	25	26	25	26	25	25	24	26	28	27	23	23	23	26	26	25	1	
		24	29	29	30	28	30	30	29	28	27	28	28	28	28	28	26	27	29	27	27	27	25	28	28	1.05	
			27	27	30	30	28	29	30	29	29	29	29	28	28	28	28	29	28	29	27	27	28	28	28	27	1.1
				26	30	31	30	30	31	28	29	29	29	29	28	28	28	28	29	29	26	28	28	28	28	27	1.15
					27	31	30	31	31	31	29	29	28	28	28	28	28	28	29	29	28	29	28	28	28	28	1.2
						28	29	29	30	31	29	28	28	27	27	27	28	29	29	28	28	29	29	28	28	28	1.25
							25	27	30	30	30	28	28	27	27	28	28	29	30	28	27	28	30	28	28	28	1.3
								26	27	29	30	28	27	28	29	28	28	29	29	29	27	28	30	30	28	28	1.35
									25	29	30	30	28	28	28	28	28	29	30	29	28	28	29	30	30	30	1.4
										25	30	31	30	28	28	29	29	29	28	28	28	28	30	30	30	30	1.45
											27	29	30	28	27	28	29	28	28	27	28	28	29	30	30	30	1.5
												27	28	29	27	28	28	30	28	27	28	28	30	30	29	1.55	
													27	28	28	28	28	29	28	29	28	29	29	29	28	1.6	
														27	28	28	28	28	28	26	26	27	29	29	29	1.65	
															27	28	28	28	27	26	26	26	27	28	28	1.7	
																26	28	28	28	27	26	27	27	27	28	1.75	
																	26	28	28	27	27	27	27	28	27	1.8	
																		26	27	28	27	27	28	28	28	1.85	
																			26	26	27	28	28	28	28	1.9	
																				25	24	26	26	27	28	1.95	
																					24	24	26	26	26	2	

Figure 7: Number of tones supported by room modes within the range A₀ to A₃ (37 semitones in total). The results are for a 150 m³ room with various dimension ratios. High numbers are best. The optimum dimension ratios found by Meissner are indicated by the letters A, B and C.

5 Frequency spacing between room modes

The third criterion applied here is solely based on the frequency distribution of the room modes. Thus, the volume and absorption properties are not involved. The idea to use the frequency spacing between room modes as a criterion was first applied by Bolt [3].

The frequency spacing index (FSI) is the normalized variance of the intervals between the low frequency room modes when arranged in order of increasing frequency. The frequency spacing index $\psi(n)$ is calculated by the formula:

$$\psi(n) = \frac{1}{n-1} \sum_{i=1}^{n-1} \left(\frac{\delta_i}{\bar{\delta}} \right)^2 \quad (3)$$

where n is the number of modes considered, and δ is the frequency difference between one mode and the previous one.

If f_1 is the frequency of the first mode, and f_n is the frequency of mode number n , the average frequency spacing is:

$$\bar{\delta} = \frac{f_n - f_1}{n - 1} \quad (4)$$

Then, $\psi(25)$ is the frequency spacing index for the first 25 room modes which is applied here.

For the 85 m³ example room from Figure 2, the first 25 room modes are shown in Table 1, sorted with increasing frequency. The frequency intervals are also displayed in the histogram in Figure 8. The calculated frequency spacing index is $\psi = 1.54$, which is very good. The frequency spacing index should be as low as possible, and the (unrealistic)

theoretical ideal is $\psi = 1$, corresponding to perfectly equal spacing of the room modes. In a real room, the lowest possible index is $\psi = 1.3$ obtained for the room dimension ratio (1:1.20:1.45), i.e. the same as the optimum A found above in section 3.

Table 1: The first 25 room modes, their frequency f_n and frequency interval δ for the example room.

n_x	n_y	n_z	f_n [Hz]	δ [Hz]
1	0	0	27.0	
0	1	0	38.7	11.7
1	1	0	47.2	8.5
2	0	0	54.0	6.8
0	0	1	57.2	3.2
1	0	1	63.3	6.0
2	1	0	66.4	3.1
0	1	1	69.1	2.7
1	1	1	74.1	5.1
0	2	0	77.3	3.2
2	0	1	78.7	1.3
3	0	0	81.0	2.3
1	2	0	81.9	0.9
2	1	1	87.7	5.8
3	1	0	89.7	2.1
2	2	0	94.3	4.6
0	2	1	96.2	1.9
3	0	1	99.1	3.0
1	2	1	99.9	0.8
3	1	1	106.4	6.5
4	0	0	108.0	1.5
2	2	1	110.3	2.3
3	2	0	112.0	1.7
0	0	2	114.4	2.5
4	1	0	114.7	0.2

Figure 8 shows the calculated frequency spacing index as function of room dimension ratios. It should be noted that these results do not depend on the volume of the room. A bad case is the cubic room (1:1:1), which has $\psi = 3.7$. Even worse is the case with dimension ratios (1:1:2), which has $\psi = 3.9$. The third bad case has dimension ratios (1:2:2) and $\psi = 3.3$. The best results are grouped in three areas that coincide almost perfectly with the three optima A, B, and C found previously using the smooth frequency response as criterium.

6 Discussion

Three different criteria have been applied for the evaluation of the acoustical consequences of the room dimension ratio. The results coincide in the optimum dimension ratios being located quite accurately around three points labelled A, B, and C in Figure 9. The results shown in Figures 2 and 8 show that the good dimension ratios form a kind of ridge that peaks at the points A, B, and C. However, this is not so clear in the results from the number of supported musical tones, see Figure 7.

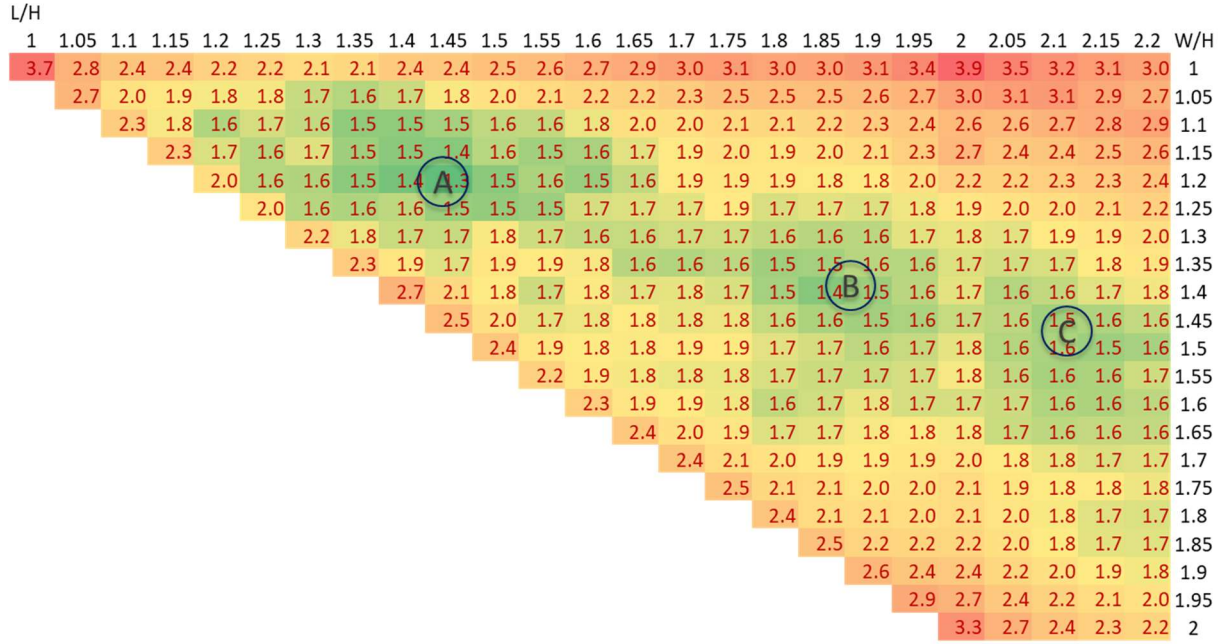


Figure 8: Frequency spacing index for the first 25 room modes $\psi(25)$ as function of the room dimension ratios. Low values shown in green colour are best and high values (in red) are worst. The optimum dimension ratios found by Meissner are indicated by the letters A, B and C.

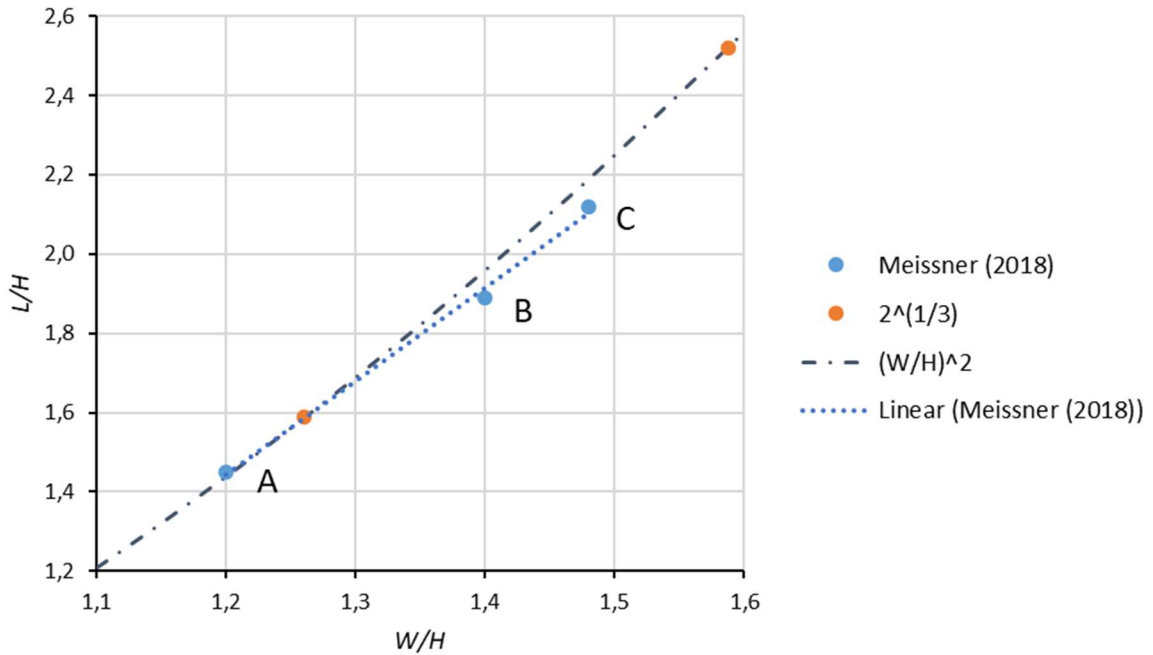


Figure 9: Comparison of optimum dimension ratios. Blue dots: The three optima found by Meissner [5] and the linear regression line given in equation (2). Orange dots: Dimension ratios following the $2^{1/3}$ rule, $(1:2^{1/3}:2^{2/3})$ and $(1:2^{2/3}:2^{4/3})$. The dashed-dotted line is for $L/W = W/H$.

Figure 9 also shows the “old” recommendations for dimension ratios following the $2^{1/3}$ rule. The dashed-dotted line indicates dimension ratios where length/width is the same as width/height. It is noted that the three optima and the linear regression line through the optima are very close to obeying the condition $L/W = W/H$.

A comparison of the results using either a smooth frequency response or the frequency spacing index is shown in Figure 10. The areas within the red contours have $\psi \leq 1.5$ and thus a good frequency distribution. The three optima based on a smooth frequency response are located within the red contours. It is observed that optimum A has the lowest (best) frequency spacing index, and the red contour around A covers a larger area than the contours around B and C.

The choice of $\psi \leq 1.5$ is rather arbitrary, and if we instead accept a value of 1.6 as sufficient, the three areas of nearly optimum dimension ratios tends to merge into a long, narrow ridge, see Figure 8. Thus, some deviation from the precise optimum dimension ratios should be allowed, and then it makes sense to use the regression line, equation (2), to find the third dimension as a function of the first two room dimensions.

In general, simple dimension ratios of 1 and 2 should be avoided. The results have shown that dimension ratios (1:1:1), (1:1:2), and (1:2:2) are particularly bad. However, the results have also shown that the dimension ratio (1:1.44:2), that is on the regression line, is not as bad as could be expected because it contains the natural number 2.

The three different quality criteria that have been applied, are compared in Table 2. The criteria are the correlation coefficient R^2 for the smoothness of the frequency response, the number of supported musical tones (#), and the frequency spacing index (FSI). The upper part of the table shows results for a selection of obviously bad dimension ratios based on natural numbers 1 to 4. The lower part of the table shows results for the three optima labelled A, B, and C, plus the ratio (1:1.44:2), which is not optimum, but appears to be surprisingly good despite the ratio 2 between length and height (labelled with *).

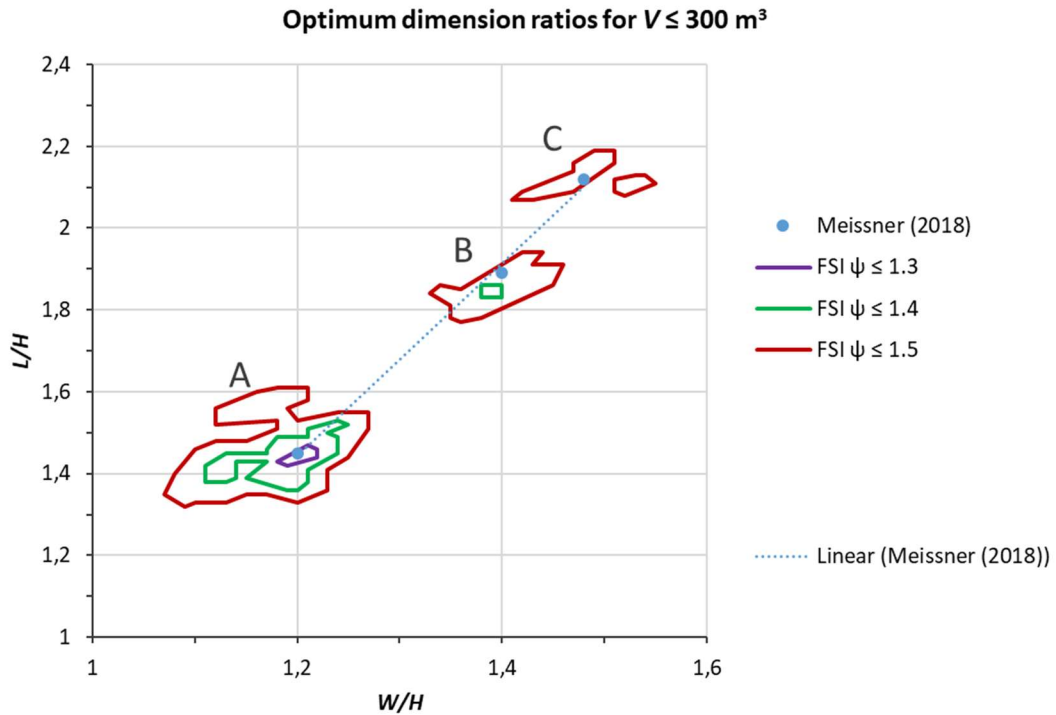


Figure 10: Comparison of two criteria for optimum dimension ratios. Blue dots: For smoothness of the frequency response. Coloured zones: For minimum frequency spacing index. Both methods point at three optimum zones, labelled A, B and C.

Table 2: Collection of bad and good dimension ratios and calculated quality criteria for a 150 m³ room. Colour code of results from very bad (red) to very good (green).

Label	W/H	L/H	H(m)	W(m)	L(m)	R^2	#	FSI
	1	1	5.3	5.3	5.3	0.73	22	3.71
	1	2	4.2	4.2	8.4	0.69	23	3.91
	1	3	3.7	3.7	11.1	0.69	26	3.00
	1	4	3.3	3.3	13.4	0.50	24	2.74
	2	2	3.3	6.7	6.7	0.68	24	3.28
	2	3	2.9	5.8	8.8	0.72	27	2.16
	2	4	2.7	5.3	10.6	0.67	26	2.70
A	1.20	1.45	4.4	5.3	6.4	0.86	31	1.33
B	1.89	1.40	3.8	5.4	7.3	0.85	29	1.51
C	2.12	1.48	3.6	5.4	7.7	0.83	30	1.54
*	1.44	2.00	3.7	5.4	7.5	0.82	28	1.68

The first observation from Table 2 is that the three criteria are in reasonably good agreement in terms of what is good and what is bad. However, the rank order of the bad rooms differs. The smoothness of frequency response points at (1:1:4) as a worst case; looking at the number of musical tones, points at the cubic room (1:1:1), and the frequency spacing index finds the case (1:1:2) to be the worst one.

The second observation is that all three criteria agree that A is the best room, closely followed by the other rooms in the group of good rooms.

A third observation is that all the good rooms have approximately the same room width, around 5.3 m, which is also the dimensions of the cubic room. This is not a big surprise, because it is just a consequence of the condition $L/W = W/H$, which was found above for rooms close to the regression line for optimum dimension ratios.

7 Suggested method for choosing room dimensions

For practical use there are two possibilities, either to aim at one of the three optimum dimension ratios, or to be less strict and choose dimensions that are nearly optimal.

For each of the optimum dimension-ratios, A, B, and C, there is a simple relation between the volume and the room height as shown in the graph in Figure 11. As an example we consider the design of a rehearsal room with the height given to be 4 m. Optimum dimension ratios can be achieved with a volume of 110 m³, 170 m³ or 200 m³, depending on which set of dimension ratios is chosen. Then it is straight forward to calculate the actual width and length. Optimum A gives (W, L) = (4.9 m, 5.8 m), optimum B gives (W, L) = (5.6 m, 7.6 m), while optimum C gives (W, L) = (5.9 m, 8.5 m), see the dots on the black lines in Figure 11.

The other possibility is to design for nearly optimal dimension ratios. This opens for a wider range of room dimensions, which can make it easier to fulfil various practical constraints. There are four simple steps:

1. Make a decision on the volume V that the room should have,
2. The width of the room should be close to $W = \sqrt[3]{V}$ (see the red curve in Figure 12),
3. Choose the room height H so that W/H is within the range 1.1 to 1.6,
4. Calculate the room length L from equation (2).

As a simple rule-of-thumb, the length/width ratio should be close to the width/height ratio. However, equation (2) is more accurate. When W/H is within the range 1.1 to 1.6 and L fulfils equation (2), the frequency spacing index is $\psi \leq 1.75$, see Figure 12.

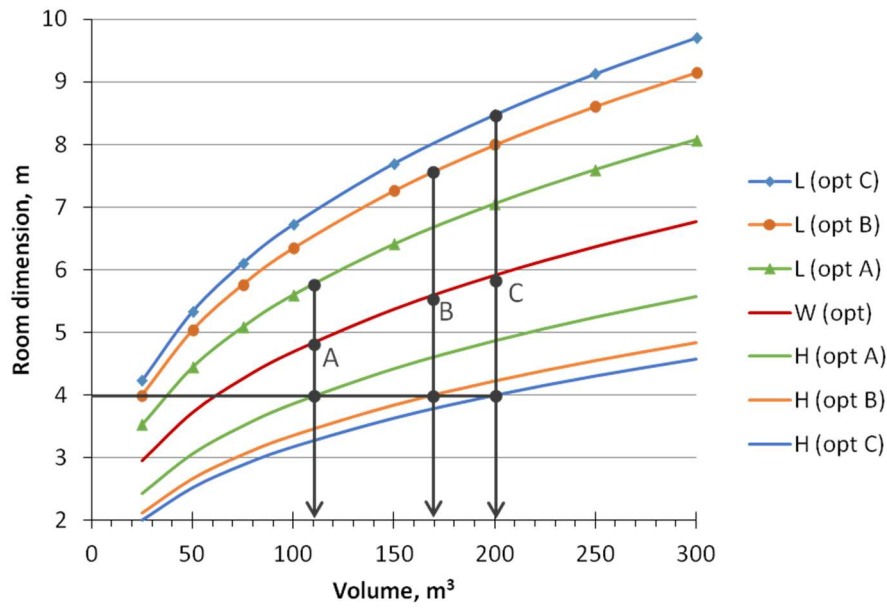


Figure 11: Relation between volume and room height, width and length for the optimum dimension ratios A, B, and C. The example shows alternative optimum volumes for a room height of 4 m, and the corresponding widths and lengths.

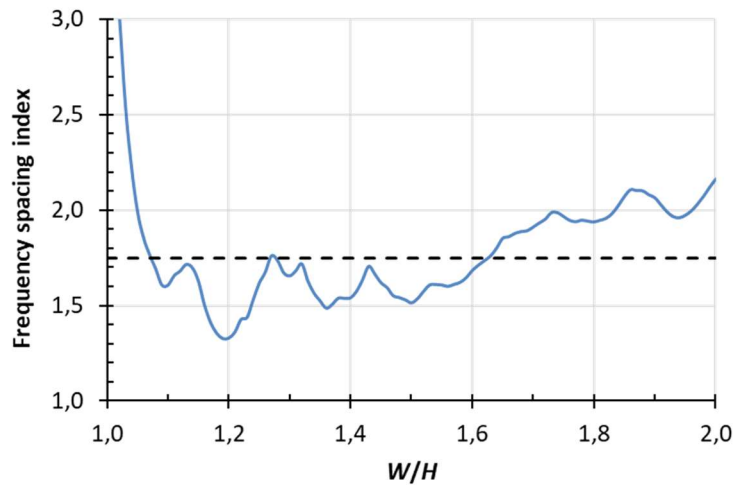


Figure 12: The frequency spacing index as function of the width-to-height ratio when the length L fulfils equation (2). The index should be as low as possible. The dashed line is the suggested maximum for nearly optimum dimension ratios.

The frequency spacing index is displayed for a wider range of the relative length and width in Figure 13. The results are divided into three groups shown in green, yellow and red colours. Green is the best with $FSI \leq 1.6$ and red is worst with $FSI > 2$. The yellow zone contains dimension ratios that can be acceptable for many uses when the acoustical requirements are not very strict. As a simple rule-of thumb this is achieved when the length-to-width ratio is within the range 1.1 to 1.6.

It is remarkable, that the length-to-width ratio is more important than any of the other dimension ratios, length-to-height, or width-to-height. This is related to the fact that length and width are assumed to be the two larger dimensions.

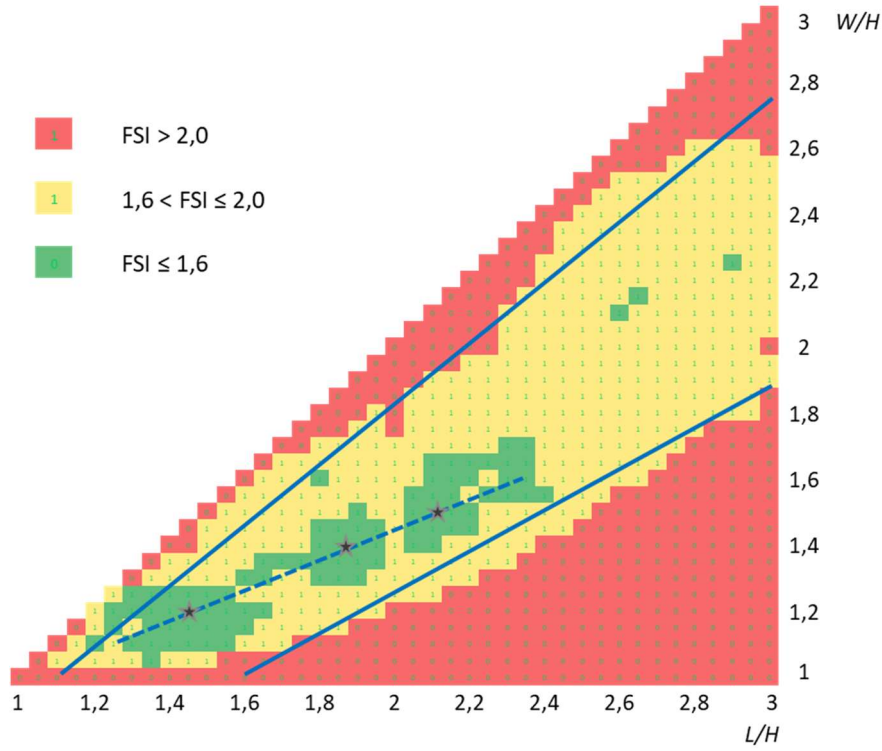


Figure 13: Calculated frequency spacing index $\psi(25)$ as function of the relative length and width. Stars: The three optima A, B, and C. Dashed line: the regression line eq. (2). Full lines: $L/W = 1.1$ and $L/W = 1.6$.

8 Conclusion

Rooms for music with volumes up to 300 m³ need careful consideration of the dimension ratio in order to offer a good acoustical support to the musical instruments. The frequency response at low frequencies should be as smooth as possible, which is closely connected to frequency distribution of the low-frequency room modes.

Three optimum dimension ratios have been found in the literature and supported by the present work. However, nearly optimum dimension ratios can be obtained within a certain range around the optimum, so good results can be obtained in practice with less strict dimension ratios.

Three different criteria have been applied to evaluate the goodness of the rooms; the smoothness of the frequency response between 20 Hz and 200 Hz, the number of musical tones between 27.5 Hz and 220 Hz supported by at least one room mode, and the frequency spacing index for the 25 lowest room modes. All three methods point at the same three optima for the room dimension ratio.

Nearly optimum dimension ratios are found close to a linear regression line that establishes a relation between L/H and W/H . Using this relation for the room design ensures a nearly optimum dimension ratio with more freedom than using only fixed optimum dimension ratios. The W/H ratio can be in the range from 1.1 to 1.6, and the L/W ratio should preferably be close to the same ratio. This implies that the width of the room should be close to $\sqrt[3]{V}$.

When the height is the smaller room dimension, it is found that the L/W ratio is more important than other dimension ratios, and that L/W should preferably be within the range 1.1 to 1.6.

References

- [1] W.C. Sabine, Reverberation. *The American Architect and The Engineering Record*, 1900. Paper No. 1 in *Collected Papers on Acoustics*. Harvard University Press, 1923. Published 1964 by Dover Publications Inc., New York.
- [2] J.E. Volkman, Polycylindrical diffusers in room acoustical design. *J. Acoust. Soc. Am.* 13, 1942, 234–243.
- [3] R.H. Bolt, Note on Normal Frequency Statistics for Rectangular Rooms. *J. Acoust. Soc. Am.* 18, 1946, 130-133.
- [4] T.J. Cox & P. D’Antonio, Determining optimum room dimensions for critical listening environments: A new methodology. *AES 110th Convention*, Amsterdam, 2001, Convention paper 5353.
- [5] M. Meissner, A Novel Method for Determining Optimum Dimension Ratios for Small Rectangular Rooms. *Archives of Acoustics* 43, 2018, 217-225.
- [6] J.H. Rindel, Modal Energy Analysis of Nearly Rectangular Rooms at Low Frequencies. *Acta Acustica united with Acustica* 101, 2015, 1211-1221.
- [7] J.H. Rindel, A Note on Modal Reverberation Times in Rectangular Rooms. *Acta Acustica united with Acustica* 102, 2016, 600-603.
- [8] R. Walker, Optimum dimension ratios for studios, control rooms and listening rooms. BBC Research Department Report, BBC RD 1993/8. <https://www.bbc.co.uk/rd/publications>