På jakt etter det musikalske øvingsrommet

(Search for the musical rehearsal room)
Abstract

• The dimensions of small rooms are important for the frequency distribution of the room modes.
• Different criteria can be applied to evaluate whether the frequency distribution is favourable; a smooth frequency response, the variance of the interval between modal frequencies, or the number of tones in the musical scale, supported by at least one of the room modes.
• The analysis leads to a practical method for choosing favourable room dimensions in a music room.
Outline

• Normal modes in a rectangular room
• Analysis of smoothness of frequency response
• Analysis of musical tones supported by room modes
• Analysis of frequency spacing between room modes
• Suggested method for choosing room dimensions
Objective criteria for the goodness of a room

• The analysis is restricted to:
  – small room volumes (up to 300 m³)
  – low frequencies (up to 220 Hz)
  – box-shaped rooms, in which the normal modes can easily be calculated

• The criteria considered are:
  – Smoothness of the global transfer function
  – Number of musical tones supported by the room modes
  – Distribution function for the separation between room modes
Normal modes of a rectangular room

Natural frequency of mode \((n_x, n_y, n_z)\): 
\[
f_n = \frac{c}{2} \sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2}
\]

Dimension ratio:
\[
1 : \frac{l_y}{l_z} : \frac{l_x}{l_z} = 1 : \frac{W}{H} : \frac{L}{H}
\]
Frequency response - Meissner (2018)

- Looked at the global frequency response curve from 20 Hz to 200 Hz
- Criterion: The curve should be as smooth as possible
- Method: Calculate the correlation coefficient for a 2\textsuperscript{nd} order polynomial.
- The correlation should be as high as possible
- NB: Result depends on the absorption of the surfaces and volume
300 m³ – Meissner (2018) Fig. 6

$$S_x = \frac{L}{H}$$
$$S_y = \frac{W}{H}$$

Optimum ratio:
$$1 : 1.2 : 1.45$$
Optimum ratios:
1 : 1.2 : 1.45
1 : 1.4 : 1.89
1 : 1.47 : 2.1
50 m³ – Meissner (2018) Fig. 7

Optimum ratios:
1 : 1.47 : 2.12
(1 : 2.55 : 3.44)

NB: The upper part of diagram is not usable due to unrealistic low Room height
Nearly optimum range

For usable room heights, the optima are located on a line.

$S_x = L/H = 2$: This is not bad, although often warned against!

The often recommended ratio $1:1.25:1.6$ is not good!
Optimum dimension ratios from Meissner (2018)

Dimension ratios that are found to produce very smooth transfer functions:

A: 1 : 1.2 : 1.45
B: 1 : 1.4 : 1.89
C: 1 : 1.48 : 2.12

Regression line is:

\[ \frac{L}{H} = 2.36 \cdot \frac{W}{H} - 1.38 \]

where

- \( L \) is room length,
- \( W \) is room width,
- \( H \) is room height.

The ratio \( W/H \) should be within the range 1.2 to 1.6.
The room as extension to a musical instrument
The room as extension to a musical instrument

$A_0 = 27.5 \text{ Hz}$

$A_3 = 220 \text{ Hz}$

The piano

Example:

150 m$^3$ room with optimum dimensions (A)

31/37 tones are matched with one or more room modes

6 tones are not well supported by the room
The room as extension to a musical instrument

\[ A_0 = 27.5 \text{ Hz} \]

\[ A_3 = 220 \text{ Hz} \]

The piano

Example:
150 m\(^3\) room with dimension ratio 1:1:1

22/37 tones are matched with one or more room modes

15 tones are not well supported by the room

22/37 tones
150 m³ rooms – Number of tones in 3 octaves

Number of tones within the range A₀ = 27,5 Hz to A₃ = 220 Hz (Max 37)

A, B and C:
Optima from Meissner (2018)

Curve for nearly optimum dimension ratios
# 50 m³ – Number of tones in 3 octaves

Number of tones within the range A₀ = 27.5 Hz to A₃ = 220 Hz (Max 37)

| L/H   | 1   | 1.05 | 1.1  | 1.15 | 1.2  | 1.25 | 1.3  | 1.35 | 1.4  | 1.45 | 1.5  | 1.55 | 1.6  | 1.65 | 1.7  | 1.75 | 1.8  | 1.85 | 1.9  | 1.95 | 2   |
|-------|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|     |     |
| W/H   | 1   | 1.05 | 1.1  |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |     |     |

- **Optima from Meissner (2018)**

Curve for nearly optimum dimension ratios
30 m³ – Number of tones in 3 octaves

Number of tones within the range A₀ = 27.5 Hz to A₃ = 220 Hz (Max 37)

A and B: Optima from Meissner (2018)

Curve for nearly optimum dimension ratios
Minimum volume for full coverage

\[ V = 1000 \text{ m}^3 \] and room dimension ratio A: 1 : 1.2 : 1.45
Frequency Spacing Index $\psi$

Definition:

$$
\psi (n) = \frac{1}{f_n - f_1} \cdot \sum_{1}^{n-1} \left( \frac{\delta^2}{\delta} \right)
$$

Where $n$ is the number of modes considered, $\delta$ is the frequency difference between one mode and the next.

The average frequency spacing is $\bar{\delta} = \frac{f_n - f_1}{n - 1}$

RMS frequency spacing is: $\delta_{rms} = \sqrt{\psi \ \bar{\delta}}$

$\psi = 1$ is ideal

$\psi = 3.5$ is worst case (a cubic room)

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Bolt (1946)

- Contour for frequency spacing index $\psi_1 < 1.5$.

Problematic to include the ratio $Y = 2$

The frequency validity range does not include the lowest room modes

The findings by Meissner (2018) are within the suggested range

Optima suggested by Meissner (2018)
# Frequency Spacing Index $\psi$ of first 25 room modes

## Frequency spacing index $\psi(25)$

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- **Worst cases**: Circled areas represent the worst cases for the frequency spacing index $\psi(25)$.
- **Best cases**: Non-circled areas represent the best cases for the frequency spacing index $\psi(25)$. 
Best – Worst (150 m³ rooms)

Histogram - frequency interval between room modes

1:1.2:1.45

1:1.4:1.85

1:1.1:1

1:1:2
Comparison of three criteria

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<th>Number of musical tones</th>
<th>Frequency spacing index</th>
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<td>Room volume</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Absorption of surfaces</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
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</tbody>
</table>

- Agreement about three optima,
  - most precise with FSI, less accurate with musical tones
Optimum dimension ratios - Comparison of two methods

Optimum dimension ratios for $V \leq 300 \text{ m}^3$

$y = 2.3558x - 1.3838$

$R^2 = 0.996$

- $\psi \leq 1.3$
- $\psi \leq 1.4$
- $\psi \leq 1.5$

Meissner (2018)
Optimum dimension ratios
- Isometric room sketches

C
1 : 1.48 : 2.12

B
1 : 1.4 : 1.89

A
1 : 1.2 : 1.45
Volume vs. room height

Example: Small ensemble room, $H \geq 3.5$ m
Suggested method for choosing room dimensions

A range of nearly optimum dimension ratios can be applied instead of a few fixed dimension ratios. This is convenient for practical use when there are constraints on room height and volume.

The formula for the regression line is:

$$L/H = 2.36 \cdot W/H - 1.38$$

where

- $L$ is room length,
- $W$ is room width
- $H$ is room height.

The ratio $W/H$ in formula should be within the range 1.2 to 1.6.
Conclusion

• The dimension ratio of a room has significant importance for the frequency distribution of room modes in small rooms.
• Several different “optimum” dimension ratios have been found in the literature.
  – However, nearly optimum dimension ratios can be obtained within a certain range around the optimum.
• Nearly optimum dimension ratios are found close to a linear regression line that establish a relation between L/H and W/H.
  – Using this relation for the room design ensures a nearly optimum dimension ratio with more freedom that using only optimum dimension ratios.
• NB: So far there is no research that show whether or not musicians prefer rooms with optimum dimension ratios.
References

• R.H. Bolt (1946). Note on Normal Frequency Statistics for Rectangular Rooms. JASA 18, 130-133.