Sirkulære lydfeller. Kanalvegger med mikroslisser

[Circular duct silencers using duct walls with micro-slits]

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Outline

- General theory of sound propagation in lined circular and annular ducts
- Special case using linings of plates with micro-slits.
- Measurement method
- Results – measurement and prediction
Dissipative type of silencers, following F.P. Mechel

Modal sound pressure

\[ p_m(r, \theta, x) = Aq_m(r, \theta)e^{-\Gamma x} \]

\[ q_m(r, \theta) = \cos(m\theta)[J_m(\varepsilon_m r) + b Y_m(\varepsilon_m r)] \quad m = 0, 1, 2, \ldots \]

Solution of the wave equation

\[
\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \Gamma^2 + k_0^2 \right] q(r, \theta) = 0
\]

Propagation coefficient gives the attenuation

\[ \text{Attenuation(dB/m)} \approx 8.69 \cdot \text{Re}(\Gamma) \]
Lined duct, i.e. without pod

Particle velocity (radial component)

\[ v_r(r, \theta, x) = \frac{j A \varepsilon_m}{k_0 Z_0} \cos(m\theta) J'_m(\varepsilon_m r) e^{-\Gamma x} \]

The impedance at \( r = h \) must match the impedance \( Z \) of the duct wall. For \( m = 0 \):

\[ \varepsilon_0 h_0 \cdot \frac{J_1(\varepsilon_0 h_0)}{J_0(\varepsilon_0 h_0)} = -j \cdot \frac{k_0 h_0}{Z_n} \]

\[ \Gamma = \sqrt{\frac{(\varepsilon_0 \cdot h_0)^2}{h_o^2} - k_0^2} \]
Now for the duct walls with micro-slits

Air-filled cavity

\[ Z_{an} = 1 \quad \text{and} \quad \Gamma_a = jk_0 \]

Micro-slit plate

\[
Z_n = \frac{1}{Z_0\sigma} \left[ Z' + j\rho_0\omega (2\Delta d_p) \right]
\]

where (with a simplified model):

\[
Z' = j\rho_0\omega d_p \left( 1 - \frac{\tan(\lambda_s / \sqrt{j})}{\lambda_s / \sqrt{j}} \right)^{-1}
\]

where \[ \lambda_s = \frac{w}{2}\sqrt{\frac{\omega\rho_0}{\mu}} \]
An interesting case?

- Hard pod
- Duct wall

Dimensions:
- 200 mm
- 100 mm
An interesting case?

In this case we need to combine the impedance of the duct wall

\[ Z_n = \frac{1}{Z_0 \varepsilon} \left[ Z' + j \rho_0 \omega (2 \Delta d_p) \right] \]

with the air around deduced from the radiation impedance of a vibrating cylinder. Assuming zero mode, we get

\[ Z_{an} = \left( \frac{Z_{rad}}{Z_0} \right)_{0,0} = j \cdot \frac{J_0(k_0 h) - jY_0(k_0 h)}{J_1(k_0 h) - jY_1(k_0 h)} \]
An interesting case?

![Graph showing transmission loss vs frequency for different conditions: Plate 2 - measured, Plate 2 - predicted, Plate 2 w/pod - measured, Plate 2 w/pod - predicted. The graph plots transmission loss (dB) against frequency (Hz) from 63 to 1000 Hz.](image-url)
How is this measurements done?

Two different two-load methods used:
2. ASTM E 2611-09
Testing set-up

![Graph showing transmission loss in dB against frequency (Hz). The graph includes a solid line labeled 'Measured', a dashed line labeled 'Predicted', and a dash-dotted line labeled 'Measured, empty duct'. The graph shows peaks and valleys across the frequency range.]

- Measured
- Predicted
- Measured, empty duct

Dimensions:
- 188 mm
- 610 mm
- 25 mm
Annular duct (1)

Particle radial velocity with pod

\[ v_r(r, \theta, x) = \frac{j A \varepsilon_m}{k_0 Z_0} \cos(m\theta) \left[ J'_m(\varepsilon_m r) + b Y'_m(\varepsilon_m r) \right] e^{-\Gamma x} \]

Knowing the impedances of duct and pod surfaces, we must solve for amplitudes A and B (=A*b)

\[ \frac{j \varepsilon_m}{k_0} \frac{A J'_m(\varepsilon_m h_{o,i}) + B Y'_m(\varepsilon_m h_{o,i})}{A J_m(\varepsilon_m h_{o,i}) + B Y_m(\varepsilon_m h_{o,i})} = \pm \frac{1}{(Z_n)_{o,i}} \]

Here again for the special case of \( m = 0 \)
Annular duct (2)

This ends up solving the equation

\[ \alpha z^2 \left[ J_1(z) Y_1(\alpha z) - J_1(\alpha z) Y_1(z) \right] \]
\[ - j U_o \alpha z \left[ J_0(z) Y_1(\alpha z) - J_1(\alpha z) Y_0(z) \right] \]
\[ + j U_i z \left[ J_1(z) Y_0(\alpha z) - J_0(\alpha z) Y_1(z) \right] \]
\[ + U_o U_i \left[ J_0(z) Y_0(\alpha z) - J_0(\alpha z) Y_0(z) \right] = 0 \quad z = \varepsilon \cdot h_0 \quad \text{and} \quad \alpha = \frac{h_i}{h_0} \]

and where the \( U \)'s contain the impedances

\[ U_o = \frac{k_0 h_o}{Z_{on}} \quad \text{and} \quad U_i = \frac{k_0 h_i}{Z_{in}} \]
Measurements – The influence of chambers
Measurements – Including a hard pod of diameter 10 cm. Two different plates.
A special case: silencer with a soft outer duct wall
Some conclusions

- Silencers with linings of micro-slitted plates may be adjusted to cover a relatively broad frequency band.
- Low frequency attenuation is substantially increased using a soft outer wall or no outer wall.
- The pressure loss of such silencers is negligible compared with conventional silencers, however not shown here.
Thank you for your attention!